

Your Signature _____

Instructions:
Please write your name on both pages.
Maximum time is 3 hours. Please stop writing when you are asked to do so.
This is a closed book exam. Calculators are permitted.
Show all your work. Correct answers with insufficient or incorrect work will not get any credit.
Please attach this sheet along with your answer sheets

Score

1.	(20)	
2.	(20)	
3.	(20)	
4.	(20)	
5.	(20)	
Total.	(100)	

Extra sheets attached(if any): _____

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1. Using Lagrange's theorem find the dimensions of the cuboid with a given surface area A ($A > 0$) but maximum possible volume. While solving the problem please appropriately justify the application of Lagrange's theorem.

2. Let $a > 0, b > 0$, $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x) = x_1 + x_2$ and $h: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by $h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$ where $h_1(x) = x_1, h_2(x) = x_3, h_3(x) = 1 - ax_1 - bx_2$ for $x \in \mathbb{R}^2$. Using Kuhn Tucker's theorem (with appropriate justification) find $x \in \mathbb{R}^2$ that maximises f on the set $\{x \in \mathbb{R}^2 : h \geq 0\}$.

3. Consider the zero-sum game with pay-off matrix

$$\begin{pmatrix} 0 & 1 & 6 \\ 4 & 2 & 0 \end{pmatrix}$$

Determine the optimal strategies for the players and the value of the game.

4. Solve the following using Simplex method:

$$\begin{array}{ll} \min & 2x_1 - x_2 \\ \text{subject to} & -x_1 + x_2 \leq 2 \\ & 2x_1 + x_2 \leq 6 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

5. Write out the dual for the following LP:

$$\begin{array}{ll} \min & -4x_1 + 2x_2 - 11x_3 + 4x_4 \\ \text{subject to} & 2x_1 - x_2 + 3x_3 - 2x_4 + x_5 = 6 \\ & x_1 + 4x_2 - x_3 - x_4 + 3x_5 \geq 4 \\ & 4x_1 + 6x_2 - x_3 - 5x_4 + x_5 \geq -8 \\ & x_1 \geq 0, x_2 \in \mathbb{R}, x_3 \geq 0, x_4 \geq 0, x_5 \in \mathbb{R} \end{array}$$

Further, verify using the complimentary slackness conditions whether $x = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \\ -3 \end{bmatrix}$ could be an optimal solution or not for the above LP.